1. Is it possible that an event is independent of itself? If so, when?

Answer:

Yes, an event can be independent of itself. This happens when the event has a probability of 0 or 1.

* **If P(A)=0P(A) = 0P(A)=0** or **P(A)=1P(A) = 1P(A)=1**, then P(A∩A)=P(A)=P(A)×P(A)P(A \cap A) = P(A) = P(A) \times P(A)P(A∩A)=P(A)=P(A)×P(A).\*\*

This satisfies the condition for independence: P(A∩A)=P(A)×P(A)P(A \cap A) = P(A) \times P(A)P(A∩A)=P(A)×P(A).

1. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.

Answer :

Yes, if AAA and BBB are independent, then their complements AcA^cAc and BcB^cBc are also independent.

**Proof:**

* Given P(A∩B)=P(A)×P(B)P(A \cap B) = P(A) \times P(B)P(A∩B)=P(A)×P(B) (independence of AAA and BBB).
* The probability of the complement events can be expressed as: P(Ac∩Bc)=P((A∪B)c)=1−P(A∪B)P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)P(Ac∩Bc)=P((A∪B)c)=1−P(A∪B) Since AAA and BBB are independent: P(A∪B)=P(A)+P(B)−P(A)×P(B)P(A \cup B) = P(A) + P(B) - P(A) \times P(B)P(A∪B)=P(A)+P(B)−P(A)×P(B) Therefore: P(Ac∩Bc)=1−(P(A)+P(B)−P(A)×P(B))P(A^c \cap B^c) = 1 - \left(P(A) + P(B) - P(A) \times P(B)\right)P(Ac∩Bc)=1−(P(A)+P(B)−P(A)×P(B)) And: P(Ac)×P(Bc)=(1−P(A))×(1−P(B))P(A^c) \times P(B^c) = (1 - P(A)) \times (1 - P(B))P(Ac)×P(Bc)=(1−P(A))×(1−P(B))

By simplifying both expressions, you can see that P(Ac∩Bc)=P(Ac)×P(Bc)P(A^c \cap B^c) = P(A^c) \times P(B^c)P(Ac∩Bc)=P(Ac)×P(Bc), confirming that AcA^cAc and BcB^cBc are independent.